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Participant

Family name : Zindulka
First name : Ondřej
Institution : Czech Technical University
Email : ondrej.zindulka@cvut.cz

Microscopic sets and Hausdorff measures

Coauthor(s)

Michael Hrušák

Abstract

A set in a metric space is *microscopic* if for any $\varepsilon > 0$ there is a sequence of sets E_n of diameters at most ε^{n+1} that covers the set. This notion was introduced by Jürgen Appell more than 20 years ago and was studied by many since then. Microscopic sets in a metric space form a σ-additive ideal with a G_δ base. They are rather small, e.g., their Hausdorff dimension is zero.

We further investigate microscopic sets, including their preservation by mappings, combinatorial structure of the ideals of microscopic sets and their cardinal invariants. We also investigate how microscopic sets are related to Hausdorff measures and as a by-product we contribute some new results to the theory of cardinal invariants of the ideals of Hausdorff measure zero.

We also notice the resemblance of microscopic sets with strong measure zero and take a look into a possible analogy of the famous Galvin-Mycielski-Solovay Theorem for microscopic sets.