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Title of the talk

On continuity in generalized topology

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Abstract

In many fields of mathematics, especially in theory of real functions, there are considered different kinds of generalized continuity. The aim of the talk is presentation of unification of properties of sets of points of these generalized continuities. The useful tool for this purpose is the notion of generalized topology introduce by Á. Császár in 2002 in [1]. It turned out that many of previously considered types of generalized continuities may be equivalently defined as a continuity in some generalized topology. The main result of this part is the following theorem, which characterize sets of points of continuity for functions defined on a generalized topological space.

Theorem 1. *Let (X, Γ) be a resolvable generalized topological space and (Y, τ) be a Moore space such that $Y^d \neq \emptyset$. For $E \subset X$ the following conditions are equivalent:*

i) $\mathcal{C}_\Gamma(f) = E$ *for some* $f\colon X \to Y$.

ii) *There exists a descending family* $(A_n)_{n\geq 1} \subset \Gamma$ *such that* $\bigcap_{n=1}^\infty A_n = E$.

The useful tool in studying this topics is the notion of topology associated with given generalized topology.

Definition 1. *Let* Γ *be a generalized topology in* X. *We will say that a topology* $\mathcal{T}$ *in* X *is associated with* Γ *if* $\mathcal{T} \subset \{B \in \Gamma\colon \forall(A \in \Gamma)\ (A \cap B \in \Gamma)\}$ *and*

$$\forall(x \in X)\, \forall(V \in \Gamma,\ x \in V)\ \big(\{x\} \cup \mathrm{int}_{\mathcal{T}}(V) \in \Gamma\big).$$

Majority of previously considered generalized continuities may be defined as a continuities in a generalized topology, which possesses an associated topology. Many interesting results can be proved for such types of continuities.

Theorem 2. *Let* (X, Γ) *be a resolvable generalized topological space,* $\mathcal{T}$ *be a topology in* X *associated with* Γ *and* (Y, τ) *be a Moore space such that* $Y^d \neq \emptyset$. *For* $E \subset X$ *the following conditions are equivalent:*

i) $\mathcal{C}_\Gamma(f) = E$ *for some* $f\colon X \to Y$.

ii) *There exists a descending family* $(B_n)_{n\geq 1} \subset \mathcal{T}$ *such that* $\bigcap_{n=1}^\infty B_n \subset E$ *and* $x \in \mathrm{int}_\Gamma(\{x\} \cup B_n)$ *for every* $x \in E$ *and* $n \geq 1$.

Very important is a generalized topology $\mathcal{Q}_{\mathcal{T}}$ which defines quasicontinuity in topological space $(X, \mathcal{T})$. For every topological space $(X, \mathcal{T})$ if Γ is a generalized topology in X such that $\mathcal{T}$ is associated with Γ then Γ is contained in $\mathcal{Q}_{\mathcal{T}}$. Moreover, we have the following characterizations:

Theorem 3. *Let* $\mathcal{T}$ *be a resolvable topology in* X *and*

$$\mathcal{Q}_{\mathcal{T}} = \{U \subset X\colon U \subset \mathrm{cl}_{\mathcal{T}}(\mathrm{int}_{\mathcal{T}}(U))\}$$

be a generalized topology defining quasicontinuity with respect to $\mathcal{T}$. *Moreover, let* (Y, τ) *be a Moore space such that* $Y^d \neq \emptyset$. *The following conditions are equivalent:*

i) $\mathcal{C}_{\mathcal{Q}_{\mathcal{T}}}(f) = E$ *for some* $f\colon X \to Y$.

ii) *There exists a descending family* $(A_n)_{n\geq 1} \subset \mathcal{Q}_{\mathcal{T}}$ *such that* $\bigcap_{n=1}^\infty A_n = E$.

iii) *There exists a descending family* $(B_n)_{n\geq 1} \subset \mathcal{T}$ *such that* $\bigcap_{n=1}^{\infty} B_n \subset E$ *and* $E \subset \mathrm{cl}_{\mathcal{T}}(\mathrm{int}_{\mathcal{T}}(B_n))$ *for every* $n \geq 1$.

Theorem 4. *Let* $(X, \mathcal{T})$ *be a perfect resolvable Baire topological space,* $E \subset X$ *and* (Y, τ) *be a Moore space such that* $Y^d \neq \emptyset$. *Then* $E \in \mathfrak{C}_{\mathcal{Q}_{\mathcal{T}}}$ *if and only if* $\mathrm{int}_{\mathcal{T}}(\mathrm{cl}_{\mathcal{T}}(E)) \setminus E$ *is a first category set in* $(X, \mathcal{T})$.

Theorem 4 generalizes results of J. S. Lipiński and T. Šalát [3]. The last part of talk will be devoted to research into path continuity with respect to a generalized topology Γ and a topology $\mathcal{T}$ which is associated with Γ. Conditions for equivalence of path continuity and continuity in generalized topology will be presented.

References

[1] Á. Császár, *Generalized topology, generalized continuity*, Acta Math. Hungar. 96 (2002), no. 4, 351–357.

[2] S. Kowalczyk, M. Turowska, *On continuity in generalized topology*, Topol. Appl. 297 (2021), 107702.

[3] J. S. Lipiński, T. Šalát, *On the points of quasicontinuity and cliqishness of functions*, Czechoslovak Math. J. 21 (96) (1971), 484–489.