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Title

Sharp Weak Type Estimates for Families of Córdoba, Soria, and Zygmud Differentiation Bases

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Abstract

Let $\mathcal{B}$ be a collection of rectangular parallelepipeds in $\mathbb{R}^3$ whose sides are parallel to the coordinate axes and such that $\mathcal{B}$ consists of parallelepipeds with sidelengths of the form $s, t, 2^N st$ (Córdoba basis), or of the form $s, 2^N/s, t$ (Soria basis), or of the form $s, 2^N s, t$ (Zygmund basis), where $s, t > 0$ and N lies in a nonempty subset S of the natural numbers. In [1, 2, 3] we prove the following:

If S is a finite set, then the associated geometric maximal operator $M_{\mathcal{B}}$ satisfies the weak type estimate

$$\left|\left\{x \in \mathbb{R}^3 : M_{\mathcal{B}} f(x) > \alpha\right\}\right| \leq C \int_{\mathbb{R}^3} \frac{|f|}{\alpha}\left(1 + \log^+ \frac{|f|}{\alpha}\right)$$

but does not satisfy an estimate of the form

$$\left|\left\{x \in \mathbb{R}^3 : M_{\mathcal{B}} f(x) > \alpha\right\}\right| \leq C \int_{\mathbb{R}^3} \phi\left(\frac{|f|}{\alpha}\right)$$

for any convex increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying the condition

$$\lim_{x \to \infty} \frac{\phi(x)}{x(\log(1+x))} = 0 .$$

Alternatively, if S is an infinite set, then the associated geometric maximal operator $M_{\mathcal{B}}$ satisfies the weak type estimate

$$\left|\left\{x \in \mathbb{R}^3 : M_{\mathcal{B}} f(x) > \alpha\right\}\right| \leq C \int_{\mathbb{R}^3} \frac{|f|}{\alpha}\left(1 + \log^+ \frac{|f|}{\alpha}\right)^2$$

but does not satisfy an estimate of the form

$$\left|\left\{x \in \mathbb{R}^3 : M_{\mathcal{B}} f(x) > \alpha\right\}\right| \leq C \int_{\mathbb{R}^3} \phi\left(\frac{|f|}{\alpha}\right)$$

for any convex increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying the condition

$$\lim_{x \to \infty} \frac{\phi(x)}{x(\log(1+x))^2} = 0 .$$

References

[1] D. Dmitrishin, P. Hagelstein, and A. Stokolos, *Sharp weak type estimates for a family of Soria bases*, *J. Geom. Anal.* Vol.32, No. 5, 11 pp., 2022.

[2] P. Hagelstein and A. Stokolos, *Sharp weak type estimates for a family of Zygmund bases*, *Proc. Amer. Math. Soc.* Vol.150, No. 5, pp. 2049–2057, 2022.

[3] P. Hagelstein and A. Stokolos, *Sharp weak type estimates for a family of Córdoba bases*, *Collect. Math.*, accepted, 2022.