A distance function equation and the good behaviour of the harmonic measure

Abstract

How does the behavior of the harmonic measure on the boundary $\partial \Omega$ of the domain $\Omega$ characterize the geometry of the boundary? For the classical setting, when the boundary $\partial \Omega$ is $n-1$-dimensional, after decades of gradual progress, the question seems to be finally settled around 2015. However, when the boundary has a smaller dimension, even the notion of the harmonic measure itself had to be reintroduced. It was done recently by David, Mayboroda, Feneuil, and other coauthors with the aid of degenerate elliptic operators, the nicest of which is

$$L_\alpha = -\text{div} D_\alpha^{-n+d+1} \nabla,$$

where $D_\alpha$ is a regularized distance function. We will discuss the key obstacle to completing the characterization of the nice geometry of the boundary for this case. It turns out to be a rather simple question concerning solutions of $L_\alpha D_\alpha = 0$. 

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