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Participant

Family name : Novosad
First name : Zoriana
Institution : Lviv University of Trade and Economics
Email : zoryana.math@gmail.com

Topological Transitive Operators on Nonseparable Space

Linear dynamics is closely related to hypercyclic and topologically transitive operators. The weighted unilateral backward shift or also known as the Rolewicz operator was one of the first hypercyclic operators found in 1960's. It was shown that no linear operator is hypercyclic on any finite-dimensional space [3]. S. Rolewich posed problem: Does an infinite-dimensional space support a hypercycic operator? A result of L. Bernal-Gonzalez [1] states that every infinite-dimensional separable Banach space supports a hypercyclic operator. The assumption of separability for the existence of a hypercyclic operator is necessary since if X is not separable, then X has no countable dense subset and hence, $Orb(T, x)$ cannot be dense in X in according to the operator T and vector x. Nonseparable spaces do not support hypercyclic operators. Since every separable Banach space admits hypercyclic and hence topological transitive operators it is natural to ask whether every nonseparable Banach space admits a topological transitive operator? Some nonseparable spaces such as $L(\ell^2)$, ℓ^∞ and $\mathcal{B}(\ell_2)$ do not support any topological transitive operator.

We research abstract shift similar operators on nonseparable function Hilbert spaces. The abstract differentiation operators and dual operators to abstract multiplication operators can be considered as abstract shift similar operators [2].

Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces, $H_n \neq \{0\}$ and not necessary separable. Let us suppose that for every n and m, H_n is isomorphic to H_m. We denote by $\ell_2(H_n) = \ell_2((H_n)_{n=0}^{\infty})$ the Hilbert space consisting of elements $x = (x_0, x_1, \ldots, x_n, \ldots)$, $x_k \in H_k$ endowed with norm $\|x\| = \left(\sum_{i=0}^{\infty} \|x_i\|^2\right)^{\frac{1}{2}}$. Let (ω_n) be a sequence of positive numbers (weights). Let us fix a sequence of isomorphisms $J_m : H_m \to H_{m-1}$, $\|J_m\| = 1$, $m \in \mathbb{N}$. An operator $T\colon \ell_2(H_n) \to \ell_2(H_n)$ will be called a *backward weighted shift (with respect to the family* (J_m)*) with weight sequence* (ω_n) if it is of the form

$$T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots).$$

Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces and $T\colon \ell_2(H_n) \to \ell_2(H_n)$ be a backward weighed shift with respect to (J_m) and with positive weight sequence (ω_n). Let us suppose that $\sup_{m\in\mathbb{Z}_+} \prod_{n=0}^{m} \|J_n^{-1}\| < \infty$. Then the operator T is topologically transitive.

References

[1] L. Bernal-González, *On hypercyclic operators on Banach spaces*, *Proc. Amer. Math. Soc.* Vol. 127, pp. 690-697, 1999.

[2] A. Zagorodnyuk, Z. Novosad, *Topological Transitivity of Shift Similar Operators on Nonseparable Hilbert Spaces*, *J. of Funct. Spaces.*, ID 9306342, 7 pages, 2021. doi: 10.1155/2021/9306342.

[3] S. Rolewicz, *On orbits of elements*, *Studia Math.* Vol. 33, pp. 17–22, 1969.