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Measure differential inclusions: existence results and minimality conditions

Abstract

The role of convergence results for integrals in the theory of differential and integral equations is well-known. Indeed, studying a large number of problems one can notice the appearance of discontinuities in the behavior of the functions, so we are lead to the idea of working with measure driven problems, i.e.

$$x(t) \in x_0 + \int_0^t F(s, x(s))dg(s), \tag{1}$$

where X is a Banach space, g is a real bounded variation function, $x_0 \in X$ and $f : [0,1] \times X \to X$. It is interesting to develop an existence theory for this kind of problems when the function g is regulated. Moreover it is important to have closure results for this problem, namely to check if when considering a sequence $(g_n)_n$ of functions converging to a function g the solutions of the equation governed by g_n is close to solutions of the equation governed by g. To this purpose, it is necessary to have a convergence result for

Stieltjes integrals and since when working with regulated functions the most appropriate integration theory is the Kurzweil- Stieltjes one, we consider a convergence theorem for the Kurzweil-Stieltjes integral. We consider also non-convex measure differential inclusions

$$\begin{aligned} &dx(t) \in G(t, x(t))d\mu_g(t), \\ &x(0) = x_0 \end{aligned} \tag{2}$$

with $x_0 \in \mathbb{R}^d$, under excess bounded variation assumptions on the velocity set $G(t, x(t))$ and make use of interesting selection principles. The map $G : [0, 1] \times \mathbb{R}^d \to P_k(\mathbb{R}^d)$ has compact possibly non-convex values and $g : [0, 1] \to \mathbb{R}$ is a left-continuous nondecreasing function whose distributional derivative (i.e. the Stieltjes measure generated by g) is denoted by μ_g. Let us remark that it is unnatural to expect the solutions to be absolutely continuous or even continuous, and so, the considered space in which the theory is developed is the space of functions of bounded variation.

References

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