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Second order differentiability and related topics in the Takagi Class

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Abstract

The Takagi function is a classical example of a continuous nowhere differentiable function, which has been studied by a large number of authors over the years. In the mid-1980s, as a generalization of the Takagi function, Hata and Yamaguti introduced a new family of functions named the Takagi class which consists of all the functions $T_w : [0, 1] \rightarrow \mathbb{R}$ defined by

$$T_w(x) = \sum_{n=0}^{\infty} \frac{w_n}{2^n} \phi(2^n x)$$

where $\phi(x) = \text{dist}(x, \mathbb{Z})$ and $w = (w_n)_n$ is a sequence satisfying $(2^{-n}w_n)_n \in \ell^1$. The Takagi class is a closed subspace of the space of continuous functions with the sup norm.

A few years later, Kôno carried out a deep study of the differentiability properties of the functions belonging to such class. More specifically, he proved that if $w \notin c_0$ then T_w is nowhere differentiable, if $w \in c_0 \setminus \ell^2$ then T_w

is not differentiable a.e. although the range of the derivative is \mathbb{R} , and finally if $w \in \ell^2$ then T_w is absolutely continuous and consequently differentiable a.e.

In this talk we will show some recent results concerning the second order differentiability of the functions belonging to the Takagi class as well as the size of the sets where these properties hold. In particular, we will characterize the set of points where these functions have a Taylor expansion of order two. Moreover, we will also characterize when they satisfy a Stepanov condition of order two at a point. Finally, we will present some interesting examples.

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