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Title of the talk

Lower Porosity on $\mathbb{R}^2$

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Abstract

The porosity of a set, defined in [3], is the notion of smallness more restrictive than nowhere density and meagerness. It can be defined in arbitrary metric space. The main idea is that we modify the "ball" definition of nowhere density by the request that the sizes of holes should be estimated. Usually, the notion of the (upper) porosity of sets is used.

Recently, the concept of porosity is used not only to study subsets of a metric space but also to compare families of functions. By applying the notion of porosity one can describe how "small" one class of function in the other is, (see [4, 5, 7]). In the study of these comparison of classes of functions, it was found that it was possible to strengthen results by using the lower porosity instead of the (upper) porosity.

We deal with the lower porosity, which has also been considered in some papers. It is known that there are big differences between the lower and the upper porosities. In [12, 11], some properties of the lower porosity in metric spaces are proved, whereas in [8], some properties of some kind of the lower porosity of subsets of $\mathbb{R}$ are presented. The first aim of the talk is to investigate new properties of the lower porosity of subsets of $\mathbb{R}^2$. It turned out that there are many distinctions between porosity on $\mathbb{R}$ and $\mathbb{R}^2$. For example, for every $U \subset \mathbb{R}$ and $x \in \mathbb{R}$, either $\underline{p}(U, x) \leq \frac{1}{2}$ or $\underline{p}(U, x) = 1$, while for every $r \in [0, 1]$ we can find a subset of $\mathbb{R}^2$ whose porosity equals r. We limit our considerations to the case of $\mathbb{R}^2$ because obtained results can be easily extended to the case of arbitrary $\mathbb{R}^n$. Properties of the lower porosity are symmetrical with properties of the (upper) porosity, but we apply cones instead of families of pairwise disjoint balls to estimate porosity.

The second aim of the talk is to describe some properties of lower porouscontinuous functions $f \colon \mathbb{R}^2 \to \mathbb{R}$. Porouscontinuous functions were introduced by J. Borsík and J. Holos in [1]. Their properties and generalizations can be found, for example, in [2, 9, 10]. Lower porouscontinuity is much stronger than (upper) porouscontinuity. In [8], some properties of lower porouscontinuity of functions $f \colon \mathbb{R} \to \mathbb{R}$ are studied. It turns out that there are big differences between lower porouscontinuity of functions defined on $\mathbb{R}$ and functions defined on $\mathbb{R}^n$. Moreover, we present similarities and differences between lower porouscontinuity and (upper) porouscontinuity. For example, we show that lower porouscontinuous functions and (upper) porouscontinuous functions have the same maximal additive class and different maximal multiplicative class.

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