STRONGLY SEPARATELY CONTINUOUS FUNCTIONS AND BOREL SUBSETS OF ℓ_p

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A notion of strongly separately continuous function of n real variables was introduced by Dzagnidze in [2] and studied in many papers (see [1, 3] and the literature given there).

Let $X_T = \prod_{t\in T} X_t$ be a product of family of sets X_t with $|X_t| > 1$ for all $t \in T$. If $S \subseteq S_1 \subseteq T$, $a = (a_t)_{t\in T}$ is a point of X_T and $x = (x_t)_{t\in S_1} \in \prod_{t\in S_1} X_t$, then x^a_S means a point $(y_t)_{t\in T}$ such that

$$y_t = \begin{cases} x_t, & t \in S, \\ a_t, & t \in T \setminus S. \end{cases}$$

In the case $S = \{s\}$ we will write x^a_s instead of $x^a_{\{s\}}$.

We say that a subset $A \subseteq X_T$ is $\mathcal{S}$*-open* if

$$\{y = (y_t)_{t\in T} \in X_T : |\{t \in T : y_t \neq x_t\}| \leq 1\} \subseteq A$$

for all $x = (x_t)_{t\in T} \in A$.

Let $X \subseteq X_T$ be an $\mathcal{S}$-open set, $\mathcal{T}$ be a topology on X and (Y, d) be a metric space. A function $f : (X, \mathcal{T}) \to Y$ is called *strongly separately continuous on* X or *an ssc-function* if

$$\lim_{x\to a} d(f(x), f(a^x_t)) = 0$$

for every $a \in X$ and $t \in T$.

In the talk we will discuss the discontinuity points set of ssc-functions and connection of ssc-functions with the box-topology of infinite products. Moreover, the problem of Baire classification of strongly separately continuous functions will lead us to a construction of an $\mathcal{S}$-open subset of ℓ_p which belongs to the α'th additive Borel class and does not belong to the α'th multiplicative Borel class.

References

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