Application of Perron Trees to Geometric Maximal Operators

We characterize the $L^p(\mathbb{R}^2)$ boundeness of the geometric maximal operator $M_{a,b}$ associated to the basis $B_{a,b}$ ($a, b > 0$) which is composed of rectangles $R$ whose eccentricity and orientation is of the form

$$(e_R, \omega_R) = \left( \frac{1}{n^a}, \frac{\pi}{4n^b} \right)$$

for some $n \in \mathbb{N}^*$. The proof involves generalized Perron trees.