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Title of the talk

Linearly continuous maps discontinuous on the graphs of twice differentiable functions

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Abstract

A function $g\colon \mathbb{R}^n \to \mathbb{R}$ is *linearly continuous* provided its restriction $g \restriction \ell$ to every straight line $\ell \subset \mathbb{R}^n$ is continuous. It is known that the set $D(g)$ of points of discontinuity of any linearly continuous $g\colon \mathbb{R}^n \to \mathbb{R}$ is a countable union of isometric copies of (the graphs of) $f \restriction P$, where $f\colon \mathbb{R}^{n-1} \to \mathbb{R}$ is Lipschitz and $P \subset \mathbb{R}^{n-1}$ is compact nowhere dense. On the other hand, for every twice continuously differentiable function $f\colon \mathbb{R} \to \mathbb{R}$ and every nowhere dense perfect $P \subset \mathbb{R}$ there is a linearly continuous $g\colon \mathbb{R}^2 \to \mathbb{R}$ with $D(g) = f \restriction P$. The goal of this talk, based on [1], is to show that this last statement fails if we do not assume that f'' is continuous. More specifically, we show that this failure occurs for every continuously differentiable function $f\colon \mathbb{R} \to \mathbb{R}$ with nowhere monotone derivative, which includes twice differentiable functions

f with such property. This generalizes a recent result from [3] of professor Luděk Zajíček and fully solves a problem from [1].

References

[1] K.C. Ciesielski, T. Glatzer, *Sets of discontinuities of linearly continuous functions*, Vol. 38, No. 2, pp. 377–389, 2013.

[2] K.C. Ciesielski, D.L. Rodríguez-Vidanes, *Linearly continuous maps discontinuous on the graphs of twice differentiable functions*, submitted, 2022.

[3] L. Zajíček, *On sets of discontinuities of functions continuous on all lines*, preprint of January 3, 2022, available at arxiv.org.