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Participant

Family name : Buczolich
First name : Zoltán
Institution : ELTE Eötvös Loránd University
Email : zoltan.buczolich@ttk.elte.hu

Title: Almost everywhere convergence questions of series of translates of non-negative functions

Coauthor(s): B. Hanson, J-P. Kahane, B. Maga, D. Mauldin, and G. Vértesy

Abstract

This talk is dedicated to the memory of Jean-Pierre Kahane

This line of research was initiated by the following question of Haight and Weizsäker:

Suppose $f : (0, +\infty) \to \mathbb{R}$ *is a measurable function. Is it true that* $\sum_{n=1}^{\infty} f(nx)$ *either converges (Lebesgue) almost everywhere or diverges almost everywhere, i.e. is there a zero-one law for* $\sum f(nx)$*?*

One can state a more general, additive version of this problem. Since $\sum_{n=1}^{\infty} f(nx) = \sum_{n=1}^{\infty} f(e^{\log x + \log n})$, that is using the function $h = f \circ \exp$ defined on $\mathbb{R}$ and $\Lambda = \{\log n : n = 1, 2, ...\}$ we can consider almost everywhere convergence questions of the series $\sum_{\lambda \in \Lambda} h(x + \lambda)$.

Suppose Λ is a discrete infinite set of nonnegative real numbers. We say that Λ is of type 1 if the series $s(x) = \sum_{\lambda \in \Lambda} f(x + \lambda)$ satisfies a zero-one law. This means that for any non-negative measurable $f : \mathbb{R} \to [0, +\infty)$ either the

convergence set $C(f,\Lambda) = \{x : s(x) < +\infty\} = \mathbb{R}$ modulo sets of Lebesgue zero, or its complement the divergence set $D(f,\Lambda) = \{x : s(x) = +\infty\} = \mathbb{R}$ modulo sets of measure zero. If Λ is not of type 1 we say that Λ is of type 2.

The exact characterization of type 1 and type 2 sets is still not known. I plan to survey some old and some more recent results related to this question.

This talk is based on several papers written at the beginning with J-P. Kahane and D. Mauldin, later with B. Hanson, B. Maga and G. Vértesy.