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On Dynamics of families of Equi-Baire one functions on metric spaces

Abstract

Let (X, ρ) be a compact metric space. The set $B_1(X, X)$ denotes the collection of all Baire one self-maps of X. In [2], we introduced the concept of Equi-Baire one as a generalization of equicontinuity for families of Baire one functions. Let (X, ρ_1), (Y, ρ_2) be two metric spaces, we say the family $\mathcal{F} \subset B_1(X, Y)$ is equi-Baire one at x_0, if for all $\epsilon > 0$ there exists a function $\delta : X \to \mathbb{R}^+$ such that

$$\rho_2(f(x), f(x_0)) \leq \epsilon \text{ for all } f \in \mathcal{F}, \text{ if } \rho_1(x, x_0) < \min\{\delta(x), \delta(x_0)\}, \text{ and}$$

$\mathcal{F}$ is equi-Baire one if for all $\epsilon > 0$, there exists a function $\delta : X \to \mathbb{R}^+$ such that for all x and y in X,

$$\rho_2(f(x), f(y)) \leq \epsilon \text{ for all } f \in \mathcal{F}, \text{ if } \rho_1(x, y) < \min\{\delta(x), \delta(y)\}.$$

First we discuss the concept of equi-B_1 and provide some intersting examples related to this concept. Let $\mathcal{K}$ be the class of compact subsets of a metric space (X, ρ), furnished with the Hausdorf metric. Here, we also study the dynamics of the limit function of a sequence of functions $\{f_n\}_{n=1}^{\infty} \subset bB_1$ and show that for a typical function $f \in B_1(I, I)$, the family $\{f^n\}_{n=1}^{\infty}$ is an equi-B_1 family and the map $\omega_f : X \to \mathcal{K}$ defined by $\omega_f(x) = \omega(x, f)$ is Baire one on I. We also show that the set of sequences that converge uniformly on X denoted by $\mathcal{F}_u(X)$, the set of sequences that are equi-B_1 denoted by $\mathcal{F}_{eq}(X)$, and the set of sequences that are point-wise convergent on X to some $f \in B_1$ denoted by $\mathcal{F}_{p.w.}(X)$ are all closed subset of $\mathcal{F}$; and $\mathcal{F}_u(X) \subsetneqq \mathcal{F}_{eq}(X) \subsetneqq \mathcal{F}_{p.w.}(X)$. Note that for two complete separable metric spaces X and Y:

• If $\{f_n\} \subset B_1(X, I)$ is a sequence that converges uniformly to f on X. Then the sequence $\{f_n\}_{n=1}^{\infty}$ is an equi-B_1 family.

• The pointwise limit of a sequence of Equi-Baire one functions from X to Y is a Baire one function.

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